An Introduction to Randomized Numerical Linear Algebra

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During these lectures we plan to introduce some central tools in randomized numerical linear algebra, study their theoretical properties, efficient implementation, and use in combination with more traditional schemes. This course alternates frontal lectures to interactive, hands-on computer lab sessions. The program language adopted throughout the course is Matlab.

A core list of topics to be discussed is the following.

- *Randomized Range Finder and Randomized SVD*: Brief recall of concepts from probability theory (expected value, variance, Gaussian matrices, etc...), presentation of the Range Finder algorithm, error bounds, its use in the Randomized SVD. Allocated time: **2h**.

- *Lab session 1*: Numerical validation of Gordon's theorem, implementation of the Randomized Range Finder algorithm and Randomized SVD. Numerical comparison with `svd` and `svds`. Allocated time: **3h**.

- *Sketching techniques*: definition of subspace embedding and randomized oblivious subspace embedding, examples of randomized oblivious subspace embedding and their use in the solution of least-squares problems. Sketch-and-solve and Sketch-and-precond approaches. Adoption of the latter in iterative methods like CGLS. Allocated time: **2h**.

- *Lab session 2*: Implementation of different sketching transformations. Numerical validation of their ε-subspace embedding property. Use of sketch-and-solve for least-squares problems: pros and cons. Use of of sketch-and-precond for least-squares problems: direct and iterative solution. Allocated time: **3h**.

- *Skecthed Krylov methods*: Crash course on Krylov methods for linear systems: Arnoldi method, FOM and GMRES, Krylov methods with incomplete orthogonalization, Krylov methods with non-standard inner products. Use of sketching as non-standard semidefinite inner product. Sketched Arnoldi relation. Skecthed GMRES and Skecthed FOM. Allocated time: **3h**.

- *Lab session 3*: Implementation of FOM and GMRES and their scketched variants. Allocated time: **3h**.