Flipping strategies for nonsymmetric Toeplitz matrices: spectral analysis and applications

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A Toeplitz matrix $T_n \in \mathbb{C}^{n\times n}$ is a matrix whose entries are constant along its diagonals. In this course, we focus on real nonsymmetric Toeplitz matrices generated by the Fourier coefficients of a given function $f \in L^1\big([-\pi,\pi]\big)$. The spectral properties of such matrices are highly nontrivial, which makes the theoretical analysis of iterative solvers for the associated linear systems, such as the GMRES method, particularly challenging.

An interesting and elegant approach to address this issue was introduced in 2015 by Pestana and Wathen, who proposed symmetrizing these linear systems by premultiplying them with the permutation matrix

$$Y_n := \begin{bmatrix} & & 1 \\ & \iddots & \\ 1 & & \end{bmatrix}_{n\times n},$$

whose effect is to reverse the order of the rows. The product Y_nT_n is then a symmetric (Hankel) matrix, allowing for a precise asymptotic spectral analysis. This transformation also enables the use of the (preconditioned) MINRES method, providing an efficient framework for solving large-scale linear systems.

We will consider both the theoretical foundations and algorithmic aspects of this flipping strategy, with particular attention to the spectral properties of the transformed matrices, the development of effective preconditioning techniques, and practical applications.